Hysteresis in model spin systems

1989 J. Phys.: Condens. Matter 1 9061

(http://iopscience.iop.org/0953-8984/1/45/030)

J. Phys.: Condens. Matter 1 (1989) 9061–9066. Printed in the UK

LETTER TO THE EDITOR

Hysteresis in model spin systems

Madan Rao, H R Krishnamurthy and Rahul Pandit
Department of Physics, Indian Institute of Science, Bangalore-560 012, India

Received 2 May 1989, in final form 3 August 1989

Abstract. We study the hysteretic response of model spin systems to periodic time-varying fields $H(t)$ as a function of the amplitude H_0 and the frequency Ω. At fixed H_0, we find conventional, squarish hysteresis loops at low Ω, and rounded, roughly elliptical loops at high Ω, in agreement with experiment. For the $O(N \to \infty)$, $d = 3$, $(\Phi^2)^2$ model with Langevin dynamics, we find a novel scaling behaviour for the area A of the hysteresis loop, of the form (valid for low fields) $A \simeq H_0^{0.66}\,\Omega^{0.33}$.

Problems in the kinetics of the first-order phase transitions have been studied extensively over the past few decades. In particular, there have been many studies of the early stages [1] (nucleation and spinodal decomposition) and late stages [2] of domain growth, after a quench from a one-phase regime to a regime where two or more phases coexist in equilibrium. By contrast, there have been hardly any studies of hysteresis [3, 4], the most commonly observed manifestation of non-equilibrium behaviour in the vicinity of a first-order phase boundary. In this Letter we initiate a systematic study of hysteresis in model spin systems.

We seek the response of a spin system to a time-varying magnetic field $H(t) = H_0 \sin(\Omega t)$. In the limit $\Omega \to 0$, we expect the magnetisation M to exhibit a discontinuity at $H = 0$ given by $M = M_{\mathrm{eq}}\,\mathrm{sgn}(H)$, where M_{eq} is the equilibrium value of M as $H \to 0+$. In the limit $\Omega \to \infty$, the spin system cannot respond to the rapidly varying magnetic field, so we expect $M(t) = M_{\mathrm{in}}$, for all time, where M_{in} is the initial value of the magnetisation of the spin system. One question of interest is: how does the discontinuity in M at $H = 0$ as $\Omega \to 0$ evolve into $M(t) = M_{\mathrm{in}}$ as $\Omega \to \infty$? The answer to this question requires a systematic study of the dependence of the shape and area of the hysteresis loop (a plot of M versus H) on the frequency Ω and the amplitude H_0. We are aware of very few detailed experimental studies of the shapes of hysteresis loops over wide ranges of the frequency [4, 5]. However, especially for ferrites [5], there are extensive data on the frequency- and amplitude-dependence of the areas of hysteresis loops and the attendant power losses. To the best of our knowledge, our work is the first statistical-mechanical attempt to explain these data. Other theories of hysteresis, either in magnets [5, 6] or bistable systems [3], do not account for spatial fluctuations of the order parameter as we do.

Our principal results are for an $O(N \to \infty)$, $d = 3$, $(\Phi^2)^2$ model with Langevin dynamics, which does not conserve any order parameter [7]:

(i) For fixed H_0 and varying Ω, the hysteresis loops show five qualitatively different asymptotic ($t \to \infty$) shapes (figure 1), which interpolate naturally between the $\Omega \to 0$

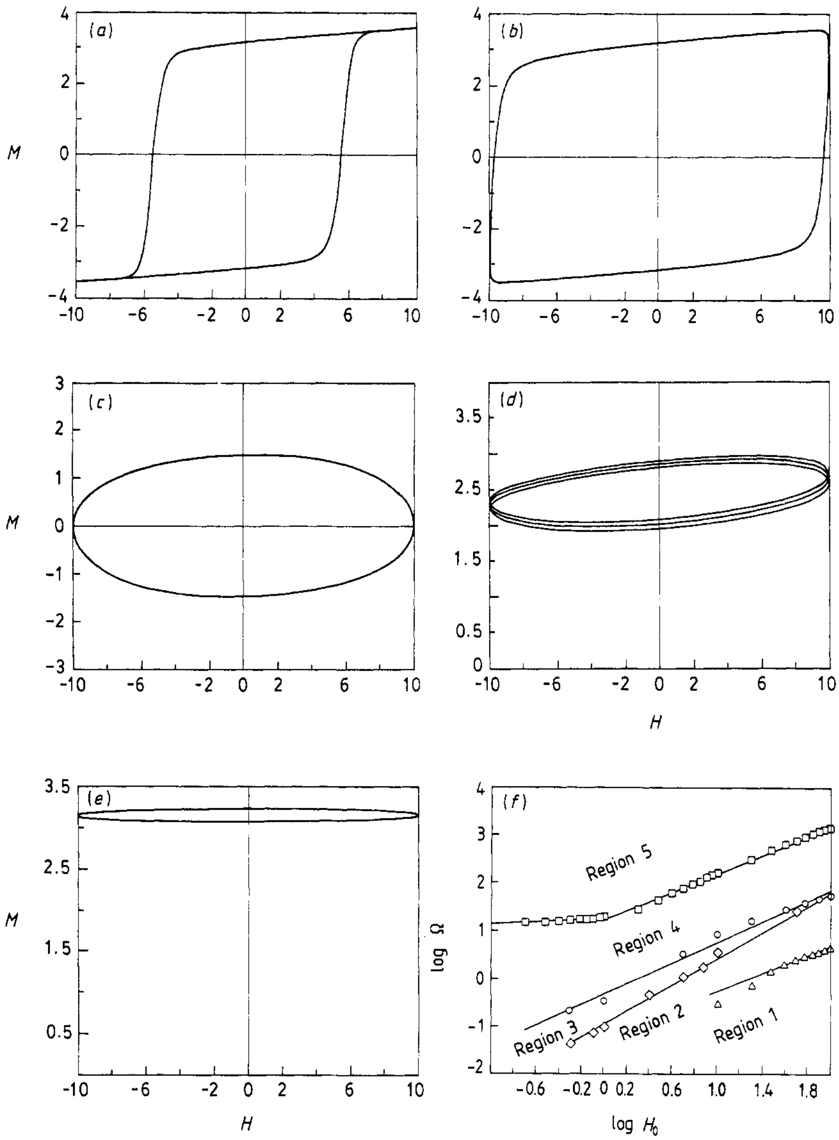

Figure 1. (*a*)–(*e*) Typical examples of the five qualitatively different hysteresis loops, and (*f*) the regions in the H_0–Ω plane where they are obtained, for the $O(n \to \infty)$ model (cf equations (1)–(3)) with $r = -10$. (*a*) $\Omega = 0.01$, (*b*) $\Omega = 0.05$, (*c*) $\Omega = 0.5$, (*d*) $\Omega = 1.2$, (*e*) $\Omega = 10$.

and $\Omega \to \infty$ behaviours discussed above. The frequency ranges in which these five shapes obtain depend on H_0: in the H_0–Ω plane we plot a stability diagram (figure 1(*f*)) which shows the regions 1, 2, 3, 4 and 5 where the five shapes are obtained asymptotically. (The criteria used for determining the boundaries between these regions are given below.)

(ii) In regions 1, 2 and 3 of figure 1(*f*), the area A of the hysteresis loop exhibits the following simple scaling behaviour: $A \simeq H_0^{\alpha}\Omega^{\beta}$, where $\alpha = 0.66 \pm 0.05$ and $\beta = 0.33 \pm 0.03$ (see figure 2). The exponents α and β are found to be independent of temperature. The scale of H_0 is set by the molecular field; that of Ω by the inverse of a

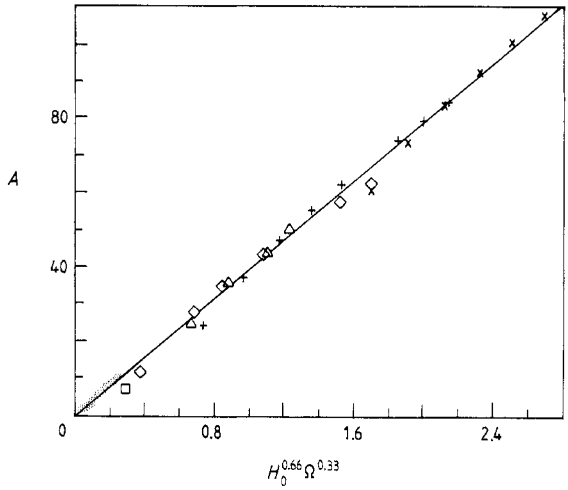

Figure 2. A scaling plot which demonstrates that the area A of the hystereis loop scales as $A \simeq H_0^{0.66}\Omega^{0.33}$. □, $\Omega = 0.01$; +, $\Omega = 0.1$; ◇, $\Omega = 0.05$; △, $\Omega = 0.0$; ×, $\Omega = 0.2$; ▽, $\Omega = 0.01$; the values of H_0 are such that all points in this figure lie in regions 1, 2, or 3 or figure 1(f); $r = -10$.

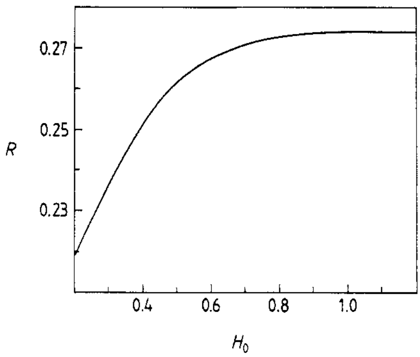

Figure 3. A plot of the ratio $R = |\tilde{M}(3\Omega)|/|\tilde{M}(\Omega)|$ versus the amplitude H_0. $\tilde{M}(w)$ is the Fourier transform of $M(t)$; $r = -10$.

microscopic relaxation time (see below). The power-law dependence on H_0 is in qualitative agreement with the well known experimental result called Steinmetz's law (where, however, $\alpha \simeq 1.6$) [5, 6].

(iii) In region 5, a simple analytical treatment yields the elliptical loop of figure 1(e); the area $A \simeq H_0^2\Omega^{-1}$ as $\Omega \rightarrow \infty$, with H_0 fixed.

(iv) The shape of the hysteresis loop can be characterised partially by studying the harmonic content of $\tilde{M}(w)$, the Fourier transform of $M(t)$. We find, in agreement with experiment, that, in regions 1, 2 and 3, $\tilde{M}(w)$ has only the fundamental and its odd harmonics. In figure 3 we plot the ratio $R = |\tilde{M}(3\Omega)|/|\tilde{M}(\Omega)|$ against H_0. R gives an estimate of the distortion of the loop (compared to an elliptical loop); thus, it is small in region 5 and increases monotonically as we go from region 5 to region 1 in figure 1(f). The saturation of R at large values of H_0 (figure 3) is in qualitative agreement with experiments on real magnets [5]. Details of our results on the response of the above spin model to magnetic-field pulses, the correlation of these results with those summarised in figure 1, and the time-dependence of the transverse correlation function will be published elsewhere [8].

We also have results for the two-dimensional Ising ferromagnet with Monte Carlo dynamics [9, 10]. These are not as extensive [10] as those for the O(N) model described above, but show an evolution of hysteresis-loop shapes similar to the evolution shown in figure 1. However, region 4 is not well defined for the Ising model: the slow drift of the loop (figure 1(*d*)) cannot be resolved because of the fluctuations inherent in a Monte Carlo simulation [10].

The O(N) model we study is specified by the Landau–Ginzburg free-energy functional

$$\beta F = \int \mathrm{d}^3 x \left(\tfrac{1}{2}(\nabla \Phi)^2 + \tfrac{1}{2} r(\Phi^2) + \frac{u}{4N}(\Phi^2)^2 - \boldsymbol{H} \cdot \Phi \right) \tag{1}$$

where $\beta = (k_B T)^{-1}$, r is proportional to $(T - T^{\mathrm{MF}})$, T^{MF} being the mean-field critical temperature for the model (1), u is positive (henceforth $u = 1$), and the magnetic field $H(t) = H_0 \sin(\Omega t)\delta_{\alpha,1}$, points along the $\alpha = 1$ direction (henceforth the longitudinal direction). $\Phi_\alpha(\boldsymbol{x}, t)$ evolve according to the Langevin equation (α goes from 1 to n)

$$\partial \Phi_\alpha(x, t)/\partial t = -\Gamma \delta(\beta F)/\delta \Phi_\alpha(\boldsymbol{x}, t) + \eta_\alpha(\boldsymbol{x}, t) \tag{2}$$

where the coefficient Γ sets the scale of time, the noise $\eta_\alpha(\boldsymbol{x}, t)$ obeys Gaussian statistics: $\langle \eta_\alpha(\boldsymbol{x}, t)\rangle = 0$ and $\langle \eta_\alpha(\boldsymbol{x}, t)\eta_\beta(\boldsymbol{y}, t_1)\rangle = 2\Gamma\delta_{\alpha\beta}\delta(\boldsymbol{x} - \boldsymbol{y})\,\delta(t - t_1)$.

Equations (1) and (2) yield an infinite hierarchy of coupled equations for the cumulants of $\Phi_\alpha(\boldsymbol{x}, t)$. In the limit $N \to \infty$, this hierarchy of equations is truncated [7], so we only have to follow the time evolution of the magnetisation $M(t) \equiv \langle \Phi_1(\boldsymbol{x}, t)\rangle$ and the transverse correlation function $C_\perp(|\boldsymbol{x} - \boldsymbol{y}|, t) \equiv \langle \Phi_\alpha(\boldsymbol{x}, t)\Phi_\alpha(\boldsymbol{y}, t)\rangle$, with $\alpha \neq 1$. The contributions of the other correlation functions to the equations for M and $C_\perp$ and $O(1/n)$, so they can be neglected in the limit $N \to \infty$. Also, as long as $M(t = 0)$ is independent of $\boldsymbol{x}$ and $C_\perp(t = 0)$ depends only on $|\boldsymbol{x} - \boldsymbol{y}|$, then $M(t)$ remains independent of $\boldsymbol{x}$ and $C_\perp(t)$ depends only on $|\boldsymbol{x} - \boldsymbol{y}|$ for all time. Thus, we have to solve the following coupled integro-differential equations [7].

$$\mathrm{d}M(t)/\mathrm{d}t = \tfrac{1}{2}[A(t)M(t) + H_0 \sin(\Omega t)] \tag{3a}$$

$$\mathrm{d}C_\perp(q, t)/\mathrm{d}t = -[q^2 - A(t)]C_\perp(q, t) + 1 \tag{3b}$$

with

$$M(t = 0) = M_{\mathrm{eq}} = \sqrt{-(r - r_c)/u}$$

$$r_c = -u/2\pi^2 \qquad C_\perp(q, t = 0) = C_{\perp\mathrm{eq}} = 1/q^2$$

where $A(t) \equiv -(r + uM^2(t) + uS(t))$, the subscript eq stands for equilibrium and

$$S = (1/2\pi^2)\int_0^1 q^2 C_\perp(q, t)\,\mathrm{d}q.$$

$C_\perp(q, t)$ is the spatial Fourier transform of $C_\perp(|\boldsymbol{x} - \boldsymbol{y}|, t)$, t is measured in units of $(2\Gamma)^{-1}$, and the upper cut-off for q is taken to be 1. We solve (3*a*) and (3*b*) numerically: we evaluate the integrals by using either Simpson's rule or Gaussian quadrature and we solve the differential equations by using Euler, Runge–Kutta, or Gear methods [11]. Thus we obtain $M(t)$ and thence the hysteresis loop in the M–H plane (figure 1). We use a fast-Fourier-transform method to obtain the Fourier transform $\tilde{M}(w)$. In this Letter we discuss only asymptotic loops ($t \to \infty$).

As stated earlier we obtain five qualitatively different loops in the five different stability regions of the H_0–Ω plane, as shown in figure 1. The boundaries separating the different regions of figure 1 should not be thought of as sharp boundaries; the changes in the shapes of the loops occur gradually. We have chosen the following criteria to determine the boundaries between the five regions given above. (i) In regions 1 and 2, as we traverse the loop in the first quadrant of the M–H plane, $\mathrm{d}^2M/\mathrm{d}H^2$ changes sign (does not change sign) if the point (Ω, H_0) lies in region 1 (region 2) of figure 1(f). (ii) Regions 2 and 3: in region 2 (region 3) M does not change sign (changes sign) as $H(t)$ passes through its maximum value H_0. (iii) Regions 3 and 4: in region 3 (region 4), the lower value of M at $H = 0$ is negative (positive), after the field H has gone through 100 cycles. (iv) Regions 4 and 5: in region 4 (region 5), the ratio $[M(t=0) - M(t=\tau)]/M(t=0)$ has a value greater than (less than) 0.01, where the ratio is evaluated at $H = 0$ and τ is the time required for 100 cycles of the field H. The boundaries between the regions obey approximate power laws with the exponents dependent on the range of H_0 [12].

In a mean-field approximation to (2) where one neglects all fluctuations, one would neglect (3b), set $S = 0$, and solve (3a) with $A(t) = -r + uM^2$. We have studied such an approximation in detail, and it also yields hysteresis loops whose shapes depend on the frequency Ω; however, a marked frequency dependence appears *only when the amplitude* H_0 *is comparable to or greater than the mean-field spinodal magnetic field*. Thus, mean-field theory is completely inadequate for the description of the frequency dependence of hysteresis loops in real magnets: the amplitudes H_0 that are accessible in laboratory magnets are many orders of magnitude lower than spinodal fields.

To compare our results with those obtained experimentally for real magnets, we must specify the scales of H_0 and t. The scale of H_0 can be set by the molecular field [13], which is typically 10^7 Oe. The scale of t is set by $(2\Gamma)^{-1}$, which is a typical microscopic relaxation time, such as the spin–lattice relaxation time, $\simeq 10^{-8}$ s [14]. Thus the frequency dependence of the shapes of the hysteresis loops should be observable at easily accessible frequencies only if H_0 is very small. While this has been noted [4], we are not aware of any experimental stability diagram such as the one we portray in figure 1(f).

It is well known that hysteretic behaviour in real magnets is determined by the dynamics of domains which are strongly affected by anisotropies, dipolar forces and defects. These effects are not included in our model studies. Hence our results may be of direct relevance only to small, monodomain magnets with very small anisotropies. A simple $1/N$ expansion is not suitable for the study of hysteresis if spin anisotropies are included, but simple decoupling approximations can be used, as we shall show elsewhere [8].

However, our results make it clear that there is much interesting physics in the phenomenon of hysteresis that is worthy of experimental and theoretical study. In particular, it would be of great interest to study the scaling behaviour (with H_0 and Ω) of the area of hysteresis loops, to find out whether this scaling is universal, and, if so, what the possible universality classes are.

We would like to thank D Dhar, C Jayaprakash, N Kumar, Y Marathe, M Phani, T V Ramakrishnan and S R Shenoy for various useful discussions. We would also like to thank the Council for Scientific and Industrial Research, the University Grants Commission, and the Department of Science and Technology (India) for support.

References

[1] See, for example, Gunton J D, San Miguel M and Sahni P S 1983 *Phase Transitions and Critical Phenomena* vol 8, ed C Domb and J L Lebowitz (London: Academic)

[2] For references see Dasgupta C and Pandit R 1986 *Phys. Rev.* B **33** 4752
Grest G S, Anderson M P and Srolovitz D J 1987 *Exxon Preprint*

[3] Agarwal G S and Shenoy S R 1981 *Phys. Rev.* A **23** 2719
Shenoy S R and Agarwal G S 1984 *Phys. Rev.* A **29** 1315
[4] Ewing J A and Klaassen H G 1983 *Phil. Trans.* A 985
[5] Smit J and Wijn H P J 1959 *Ferrites* (Philips Technical Library) ch 16
[6] Bozorth R M 1951 *Ferromagnetism* (New York: Van Nostrand) ch 17
[7] Mazenko G F and Zannetti M 1985 *Phys. Rev.* B **32** 4565
[8] Rao M, Krishnamurthy H R and Pandit R to be published
[9] Binder K 1979 *Monte Carlo Methods in Statistical Physics* ed. K Binder (Berlin: Springer) ch 1
[10] In our Ising Monte Carlo simulations, for numerical convenience, we did not use a sinusoidally varying magnetic field. Instead, we used a field that increases linearly (in small steps) from $-H_0$ to H_0, and then repeats this periodically. Because of fluctuations, every time $H(t)$ goes through one cycle, a slightly different hysteresis loop is traced out. Many loops must be averaged over (with different random number seeds as well) to obtain a mean hysteresis loop. Since this averaging takes a long time, our data for the Ising model are not as extensive as our data for the O(n) model.
[11] Press W H, Flannery B P, Teukolsky S A and Vetterling W T 1986 *Numerical Recipes* (Cambridge: CUP) ch 15
[12] The boundary between region 1 and region 2 is given by $\Omega \simeq H_0^{0.95}$ for $10 \leqslant H_0 \leqslant 100$. The boundary between region 2 and region 3 is $\Omega \simeq H_0^{1.35}$ for $10 \leqslant H_0 \leqslant 100$ and $\Omega \simeq H_0^{1.5}$ for $0.1 \leqslant H_0 < 10$. Region 3 and region 4 are divided by the boundary whose equation is $\Omega \simeq H_0^{1.085}$ for $0.1 \leqslant H_0 \leqslant 100$. The region 4–region 5 boundary is given by $\Omega \simeq H_0^{0.97}$ for $1 \leqslant H_0 \leqslant 100$ and $\Omega \simeq H_0^{0.13}$ for $0.1 \leqslant H_0 \leqslant 1$.
[13] Kittel C 1949 *Rev. Mod. Phys.* 1949 **21** 541
The molecular field $H_{mf} = k_B T_c/\mu_B$ where T_c is of the order of 10^3 K (for iron).
[14] Bloembergen N and Wang S 1954 *Phys. Rev.* **93** 72